

Suppose lossless medium. ( $\sigma = 0$ , both)

The net  $\vec{E}$  field is ①

$$\vec{E}_{is} + \vec{E}_{rs} = \hat{x} E_i e^{-\alpha_1 z} e^{-j\beta_1 z} + \hat{x} \Gamma E_i e^{+\alpha_1 z} e^{+j\beta_1 z}$$

$$\sigma_1 = 0 \quad \therefore \quad \alpha_1 = 0$$

The physical field in medium 1 becomes

$$\text{Re}[\hat{x} E_i (e^{-j\beta_1 z} e^{j\omega t} + \Gamma e^{j\beta_1 z} e^{j\omega t})]$$

$$\hat{x} E_i \text{Re}[\cos(\omega t - \beta_1 z) + j \sin(\omega t - \beta_1 z) + \Gamma \cos(\omega t + \beta_1 z) + \Gamma j \sin(\omega t + \beta_1 z)]$$

If  $\sigma_1 = \sigma_2 = 0$  then  $\alpha_1 = \alpha_2 = 0$

then gamma is real ( $\Gamma$ )

$$\hat{x} E_i [\cos(\omega t - \beta_1 z) + \Gamma \cos(\omega t + \beta_1 z)]$$

Suppose  $\Gamma = -1$  ( $n_1 \ggggg n_2$ )

$$\hat{x} E_i (\cos(\omega t - \beta_1 z) - \cos(\omega t + \beta_1 z))$$

can be 0 when

$$\omega t - \beta_1 z = \omega t + \beta_1 z \pm 2\pi n$$

(dielectric to dielectric boundary)

### Dielectric - Conductor Boundary



incident  
med

$$\sigma_1 = 0$$

$$\alpha_1 = 0$$



$$\sigma_2 = \infty$$

$$\alpha_2 = \infty$$

$$\eta = 0$$

$$\vec{E}_i = \hat{x} E_i e^{-\alpha_2 z} e^{-j\beta_2 z}$$

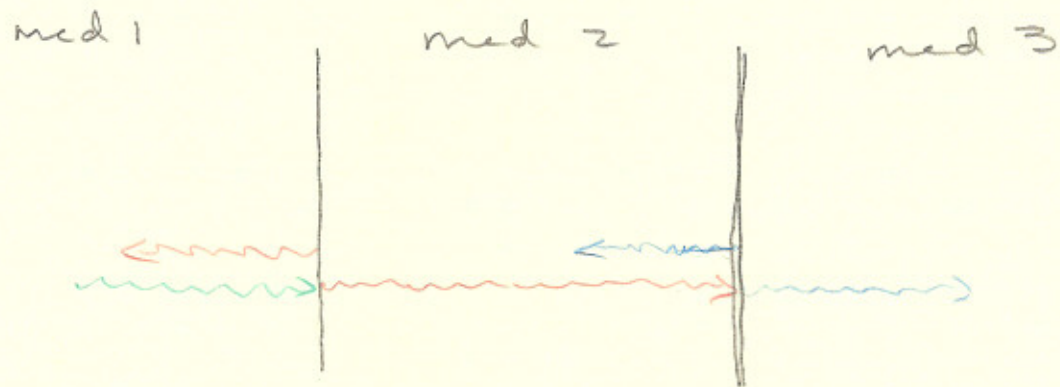
note all electric field is reflected.

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = 0 \quad (\text{nothing is trans.})$$

$$\Gamma = -1 \quad (\text{everything is reflected})$$

note if  $\Gamma$  is greater than zero then the electric field is positive wrt. the  $E_i$ , if  $\Gamma < 0$  then neg wrt.

## Multiple interfaces



$E_i$  is net incident field ...

$E_r$  is net reflected field in.

in med ①  
(includes back scatter)

$E_z^+$  is the net field towards right

$E_z^-$  is the net field towards left

med ②

$E_{zt}$  is the net transmitted field

med ③

$$\vec{H}_{ci} = \frac{1}{\eta_{c1}} (\hat{z}) \times \hat{x} (E_i e^{-\alpha_1 z} e^{-j\beta_1 z})$$

$$\vec{H}_{ri} = \frac{1}{\eta_{c1}} (\hat{z}) \times \hat{x} (E_i e^{+\alpha_1 z} e^{+j\beta_1 z})$$

$$\vec{H}_{net} = \frac{1}{\eta_{c1}} \hat{y} (E_i e^{-\alpha_1 z} e^{-j\beta_1 z} - E_r e^{+\alpha_1 z} e^{+j\beta_1 z})$$

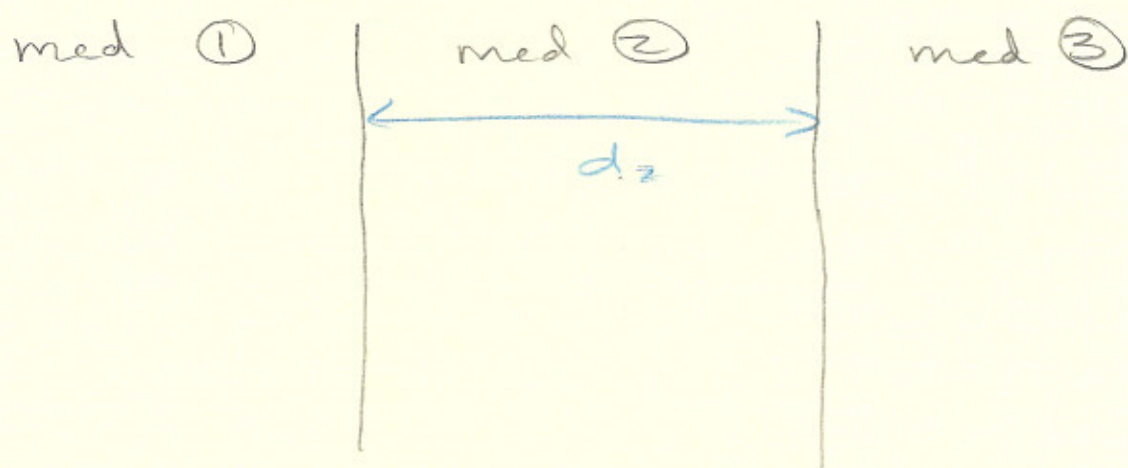


$$\vec{H}_2 = \frac{1}{\eta_{c2}} \hat{y} (E_2^+ e^{-\alpha_2 z} e^{-j\beta_2 z} + E_2^- e^{+\alpha_2 z} e^{+j\beta_2 z})$$

$$\vec{E}_{3t} = \hat{x} E_3^+ e^{-\alpha_3 z} e^{-j\beta_3 z}$$

$$\vec{H}_{3t} = \frac{1}{\eta_{c3}} \hat{y} E_3^+ e^{-\alpha_3 z} e^{-j\beta_3 z}$$

4 eqs for 5 unknowns, but we have  $E_i$



$$\frac{E_r}{E_i} = \frac{(\eta_{c2} - \eta_{c1})(\eta_{c2} + \eta_{c3}) + e^{-2\alpha_2 d_z} (\dots)}{(\eta_{c1} + \eta_{c2})(\eta_{c2} + \eta_{c3}) + e^{-2\alpha_2 d_z} (\dots)}$$

$$\frac{E_3^+}{E_i} = \dots$$

See distributed notes for these and more (egs 10.13 → 10.16)